Chemical Reaction, Radiation and Heat Source Effects on Unsteady MHD Blood Flow Over a Horizontal Porous Surface in the Presence of an Inclined Magnetic Field.

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Abstract— In this paper, an analysis of chemical reaction, heat source and thermal radiation effects on unsteady blood flow through a plate channel that is parallel and horizontal with an inclined magnetic field in a medium that is porous and saturated is considered. The governing equation that is non-linear higher partial differential equation are converted to ordinary differential equations using dimensionless variables to dimensionless equations, which is analytically solved with applied boundary conditions using velocity, concentration and temperature for the functions of y and t. Different parameters had effects on the blood flow temperature and concentration with the results discussed and illustrated graphically.

Index Terms— Chemical Reaction, Radiation, Heat Source, Suction, Inclined Plate, Porous Surface, Magneto Hydrodynamic (MHD) Blood Flow.

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1. Introduction

MHD blood flow has rich application in sciences and biomedical engineering since it deals with flow of fluid that conducts electricity in the magnetic field. Certain mediums such as solid bodies are porous making them to have pores. Porosity has applications in bio-fluids and body component such as bones, kidneys, lungs, tissues and body organs. The heart pumps blood which creates pressure required for blood circulation which provides oxygen and nutrients for lungs, heart, liver, brain, kidney etc.

The first study on electromagnetic flow and its application to measurement of blood flow was done by Kolin [7] and then extended by Korchevskii [8] who did a study on cardiovascular disease treatment using MHD device to reduce the velocity flow. Certain ailments such as cancer and tumor can also be treated with MHD devices. Isreal-Cookey [6] investigated the influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction while Mebine [10] did a study on the thermal solutal MHD flow with radiative heat transfer over oscillating plate in a fluid that is chemically active. Prakash [12] studied the effect of heat sources on MHD blood flow through a bifurcated artery, Tripathy [4] did a mathematical model for blood flow through an artery that is inclined under an inclined magnetic field.

Neetu Srivastava [11] did an analysis of the flow characteristics of the blood flowing through an inclined tapered porous artery with mild stenosis under the influence of an inclined magnetic field. Tzirtzilakis [5] did a mathematical model for blood flow in a magnetic field. Blessy Thomas [1] studied a review on blood flow in a human arterial systems carrying out simulations of blood flow with a view in structure and function of arteries and veins using models for blood flow and fluid structures.

Latha [9] did a study on MHD unsteady blood flow through porous medium in a parallel plate channel, velocity profile for unsteady blood flow through a circular tube inclined in the presence of a magnetic field was studied by Vincent [13].

Tripathy [3] studied the effect of variable viscosity on MHD inclined Arterial blood flow with chemical reaction while Asma Khalid [1] studied MHD blood flow in a porous medium with CNTS and thermal analysis.

This paper will therefore look at how high blood pressure can be treated with magnetic field and how cancer cells can be destroyed with increased radiation and increased chemical reaction

2. Mathematical Formulation

The blood flowing in the medium that is porous is a Newtonian fluid that has a constant viscosity and is incompressible. The mathematical model will be considering the effect of an inclined magnetic field applied on the blood flow. In the equations above, u and v are the velocities, x and y are the Cartesian coordinates, α is the angle of inclined magnetic field, t is the time, θ is the temperature and g is the acceleration due to gravity. B_0 is the constant magnetic field, K is the porosity parameter, C_p is the specific heat capacity and M is the magnetic parameter. Pr is the Prandtl number, ρ is the density of the blood, β_T is the coefficient of volume expansion due to temperature and β_C is the coefficient of volume expansion due to concentration, δ_c is the electrical conductivity, K_r is the chemical reaction parameter, R is the radiation parameter and Sc is the Schmidt number, L is the diameter of the porous medium, S is the heat source and m is the rate of mass flow.

Thus the flow equations are as follows.

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} + v\frac{\partial u'}{\partial y'} + \frac{1}{\rho}\frac{\partial p'}{\partial x} = v\frac{\partial^2 u'}{\partial y'} - \left[\frac{\delta_c B_0^2 \cos^2\alpha}{\rho} + \frac{v}{k}\right]u + gB_T\theta' + gB_C C' \quad (2)$$

$$\frac{\rho C_p}{K_T}\left[\frac{\partial T'}{\partial t'}\right] + v\frac{\partial T'}{\partial y'} = \frac{\partial^2 T'}{\partial y'^2} + \frac{Q}{K_T}\theta' - \frac{\partial q_r'}{\partial y'} \quad (3)$$

$$\frac{\partial C'}{\partial t'} + v\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - K_r'(C' - C_\infty) \quad (4)$$

The thermal radiation heat flux q using Rosseland's approximation is expressed as

$$q_r' = -\frac{4\delta'}{3k'}\frac{\partial T'}{\partial y'} = -\frac{4\delta'}{3k'}\nabla T'$$

Where δ' is the Stefan – Boltzmann constant and k' is the rosseland mean absorption coefficient. Assuming that the temperature differences within the flow are sufficiently small that T' may be expressed as a linear function of temperature

$T' = 4T_O^3 T - 3T_O'$ This implies that

$$q_r' = -\frac{16\delta' T_O^3}{3k'}\frac{\partial T}{\partial y} \quad (5)$$

The higher order terms of the expansion (tailors series expansion) i.e T_O' is neglected

The non-dimensional quantities are introduced in order to write the governing equations and boundary conditions in dimensionless form.

$$y = \frac{y'}{h}; x = \frac{x'}{h};\ u = \frac{u'm}{2\rho h};\ t = \frac{\mu t'}{\rho h^2};\ \theta = \frac{\theta' 2\rho^2 h^3}{\mu m};\ C = \frac{C' 2\rho^2 h^3}{\mu m}; h(x,t) = (\partial p'/\partial x')/(\mu m/2\rho^2 h^3); M = \frac{\delta_c B_0^2 L^2}{\mu}; K = \frac{K_r' L^2}{v}; R = \frac{16\delta' T_0^2}{3k'k}; v = \frac{\mu}{\rho}; P_r = \frac{\mu C_p}{K_T}; S_c = \frac{v}{D}; H = \frac{QL^2}{K_T} \quad (6)$$

The continuity, momentum, energy and diffusion equation in dimensionless form is written as

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (7)$$

$$\frac{\partial u}{\partial t} + L\frac{\partial u}{\partial y} + h = \frac{\partial^2 u}{\partial y^2} - \left(M\cos^2\alpha + \frac{L^2}{K}\right)u + g\beta_T\theta + g\beta_c C \quad (8)$$

$$P_r\frac{\partial \theta}{\partial t} + P_r L\frac{\partial \theta}{\partial y} = (P_r + R)\frac{\partial^2\theta}{\partial y^2} + S\theta \quad (9)$$

$$S_c\frac{\partial C}{\partial t} + S_c L\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} - S_c K_r C \quad (10)$$

3. Solution to the Problem

The corresponding boundary conditions in non-dimensional form are:

$$u = e^{-\lambda^2 t}, \theta = e^{-\lambda^2 t}, C = e^{-\lambda^2 t}, \quad at\ y = -1$$
$$u = 0, \theta = 0, C = 0, \quad at\ y = 1 \quad (11)$$

The solutions of equations (7) – (10) will become

$$u(y,t) = F(y)e^{-\lambda^2 t} \quad (12)$$

$$v(y,t) = A(y)e^{-\lambda^2 t} \quad (13)$$

$$\theta(y,t) = G(y)e^{-\lambda^2 t} \quad (14)$$

$$C(y,t) = H(y)e^{-\lambda^2 t} \quad (15)$$

Substituting equation (12) – (15) into equation (7) – (10) we will obtain

$$\frac{\partial^2 F}{\partial y^2} - L\frac{\partial F}{\partial y} + \left(\lambda^2 - M\cos^2\alpha + \frac{L^2}{K}\right)F = -g\beta_T\theta - g\beta_c C + h \quad (16)$$

$$(P_r + R)\frac{\partial^2 G}{\partial y^2} - P_r L\frac{\partial G}{\partial y} + (S + P_r\lambda^2)G = 0 \quad (17)$$

$$\frac{\partial^2 H}{\partial y^2} - S_c L\frac{\partial H}{\partial y} + (S_c\lambda^2 - S_c K_r)H = 0 \quad (18)$$

$$F = 1; G = 1;\ H = 1; \quad at\ y = -1$$
$$F = 0; G = 0;\ H = 0; \quad as\ y = 1 \quad (19)$$

Substituting equation (15) – (20) into equation (12) – (14) we obtain the velocity, temperature and concentration profiles respectively as

$$u(y,t) = [d_1 e^{m_3 y} + d_2 e^{-m_3 y} - d_3 e^{-m_2 y} - d_4 e^{-m_2 y} - d_5 e^{-m_1 y} - d_6 e^{m_1 y} - d_6 e^{m_1 y} + d_7]\ e^{-\lambda^2 t} \quad (20)$$

$$\theta(y,t) = [c_1 e^{m_2 y} + c_2 e^{-m_2 y}]e^{-\lambda^2 t} \quad (21)$$

$$C(y,t) = [b_1 e^{m_1 y} + b_2 e^{-m_1 y}]e^{-\lambda^2 t} \quad (22)$$

4. Results and Discussion

In this session we will discuss the effect of different parameters on the Blood flow, temperature, and concentration. In figure 4.1, we will observe that an increase in the radiation causes an increase in the velocity profile and then a decrease in the blood

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temperature in figure 4.9. When the Prandtl number increases in figure 4.2, the velocity profile of blood flow increases and then the temperature of the blood in figure 4.10 declines as a result of increased Prandtl number.

We observed that in figure 4.3, an increase in heat source causes the blood flow to reduce but increased heat source increases the blood temperature in figure 4.11. Figure 4.4 shows that an increase in the chemical reaction causes a reduction in the blood flow and a reduced diffusion of blood in figure 4.13. Increase in porosity in figure 4.5 increases the blood flow near the wall and then reduces the blood flow near the boundary layer. In figure 4.6, an increase in the magnetic field reduces the blood flow as a result of Lorentz force been introduced causing reduction in the viscosity of the blood while an increase in the angle of inclination reduces the blood flow in figure 4.7. The increase in time in figure 4.8, figure 4.12 and figure 4.16 reduces the blood flow, blood temperature and diffusion profile.

In figure 4.14, an increase in Schmidt number reduces the concentration or diffusion profile while an increase in the diameter of the porous medium causes a reduction of the diffusion profile in figure 4.15

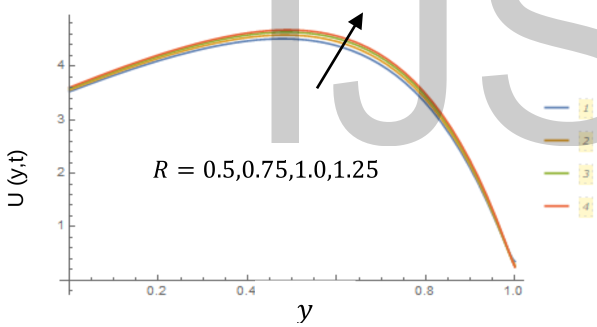

Figure 4.1 **Velocity Profile with Variation of Radiation Parameter *R***
$Pr = 0.5, L = 0.5, S = 1, \lambda = 0.5, Sc = 1, Kr = 0.8, K = 0.1, M = 0.5, \alpha = 10, h = 0.5, g = 9.81, \beta 1 = 0.5, \beta 2 = 0.5, t = 1$

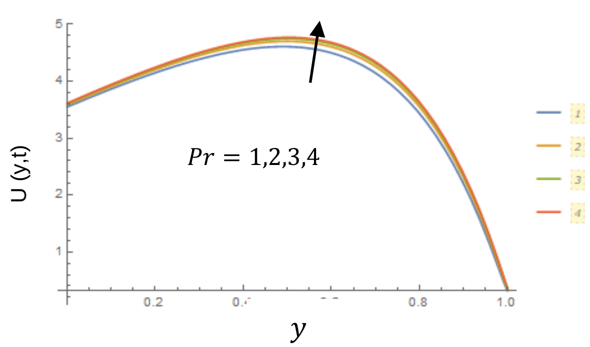

Figure 4.2 **Velocity Profile with Variation of Prandtl number Parameter Pr**
$R = 0.5, L = 0.5, S = 1, \lambda = 0.5, Sc = 1, Kr = 0.8, K = 0.1, M = 0.5, \alpha = 10, h = 0.5, g = 9.81, \beta 1 = 0.5, \beta 2 = 0.5, t = 1$

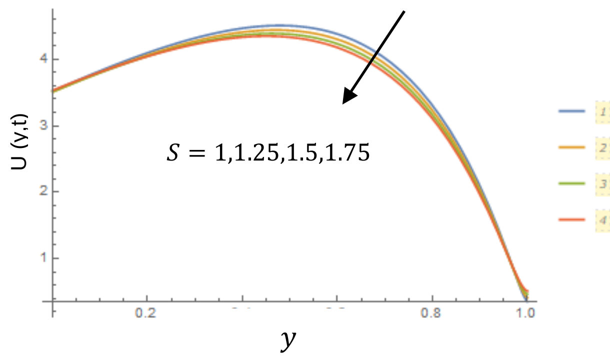

Figure 4.3 **Velocity Profile with Variation of Heat Source Parameter S**
$R = 0.5, Pr = 0.5, L = 0.5, \lambda = 0.5, Sc = 1, Kr = 0.8, K = 0.1, M = 0.5, \alpha = 10, h = 0.5, g = 9.81, \beta 1 = 0.5, \beta 2 = 0.5, t = 1$

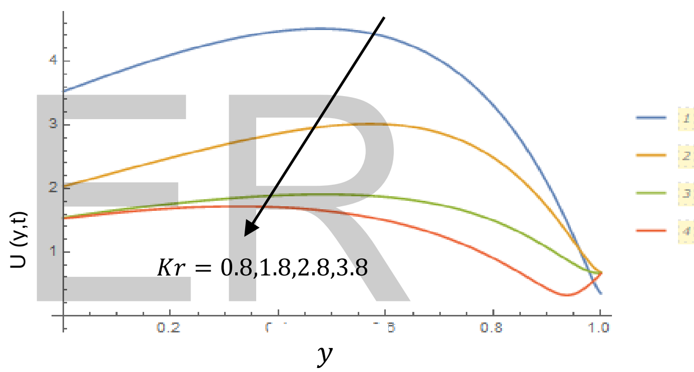

Figure 4.4 **Velocity Profile with Variation of Chemical reaction Parameter Kr**
$R = 0.5, Pr = 0.5, L = 0.5, S = 1, \lambda = 0.5, Sc = 1, K = 0.1, M = 0.5, \alpha = 10, h = 0.5, g = 9.81, \beta 1 = 0.5, \beta 2 = 0.5, t = 1$

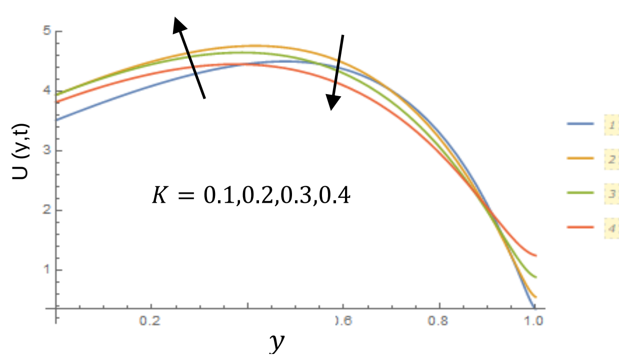

Figure 4.5 **Velocity Profile with Variation of Porosity Parameter for K**
$R = 0.5, Pr = 0.5, L = 0.5, S = 1, \lambda = 0.5, Sc = 1, Kr = 0.8, M = 0.5, \alpha = 10, h = 0.5, g = 9.81, \beta 1 = 0.5, \beta 2 = 0.5, t = 1$

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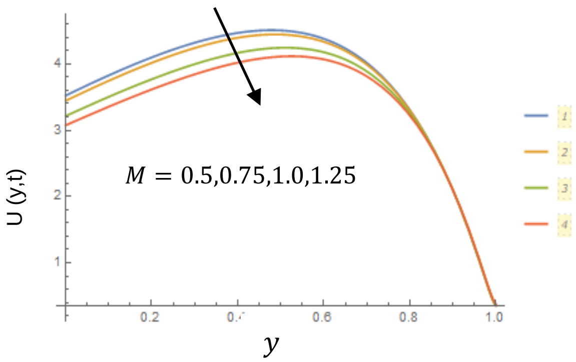

Figure 4.6 **Velocity Profile with Variation of Magnetic Field Parameter M**
$R = 0.5, Pr = 0.5, L = 0.5, S = 1, \lambda = 0.5, Sc = 1, Kr = 0.8, K = 0.1, M = 0.5, \alpha = 10, h = 0.5, g = 9.81, \beta 1 = 0.5, \beta 2 = 0.5, t = 1$

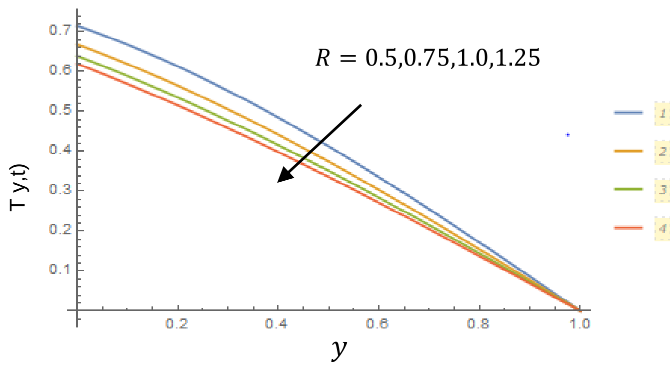

Figure 4.9 **Temperature Profile with Variation of Radiation Parameter *R***
$Pr = 0.5, L = 0.5, S = 1, \lambda = 0.5, t = 1$

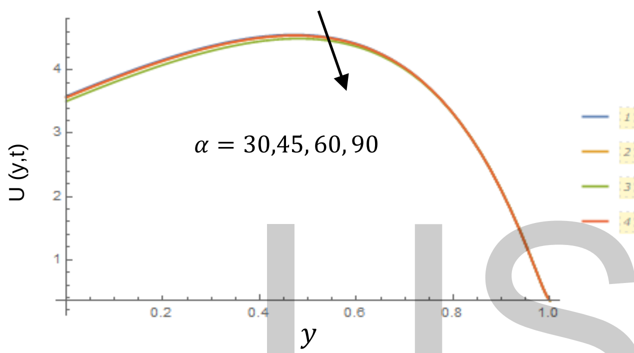

Figure 4.7 **Velocity Profile with Variation of angle of inclination Parameter α**
$R = 0.5, Pr = 0.5, L = 0.5, S = 1, \lambda = 0.5, Sc = 1, Kr = 0.8, K = 0.1, M = 0.5, \alpha = 10, h = 0.5, g = 9.81, \beta 1 = 0.5, \beta 2 = 0.5, t = 1$

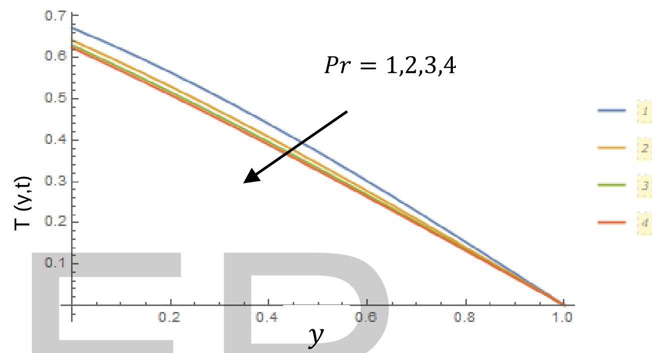

Figure 4.10 **Temperature Profile with Variation Prandtl number Parameter *Pr***
$R = 0.5, L = 0.5, S = 1, \lambda = 0.5, t = 1$

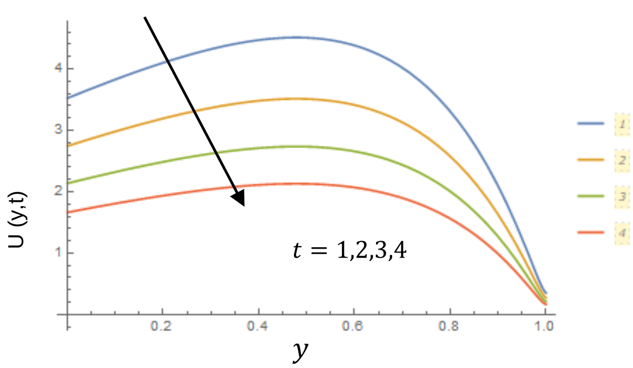

Figure 4.8 **Velocity Profile with Variation of time Parameter *t***
$R = 0.5, Pr = 0.5, L = 0.5, S = 1, \lambda = 0.5, Sc = 1, Kr = 0.8, K = 0.1, M = 0.5, \alpha = 10, h = 0.5, g = 9.81, \beta 1 = 0.5, \beta 2 = 0.5$

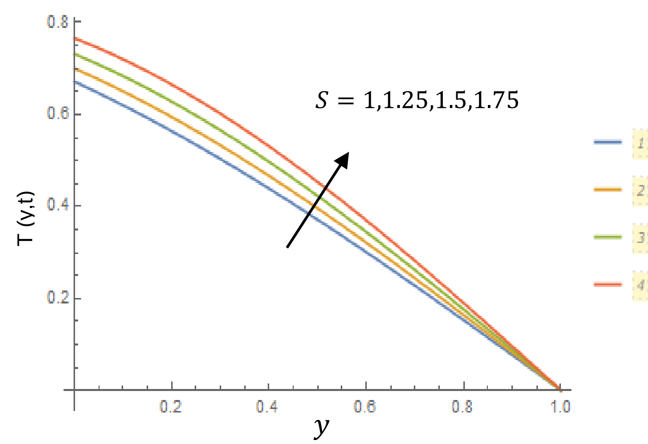

Figure 4.11 **Temperature Profile with Variation of Heat Source Parameter *S***
$R = 0.5, Pr = 0.5, L = 0.5, \lambda = 0.5, t = 1$

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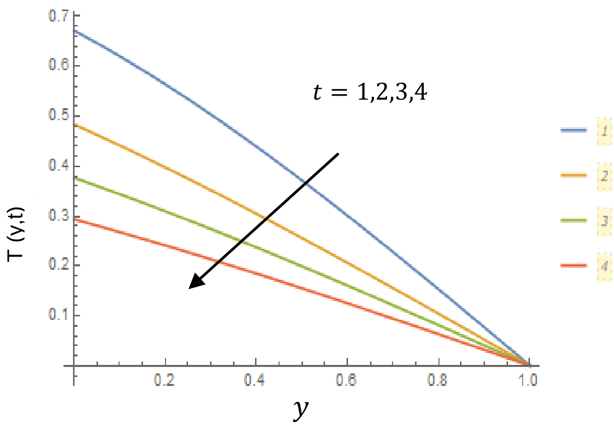

Figure 4.12 **Temperature Profile with Variation of time Parameter t**
$R = 0.5, Pr = 0.5, L = 0.5, S = 1, \lambda = 0.5$

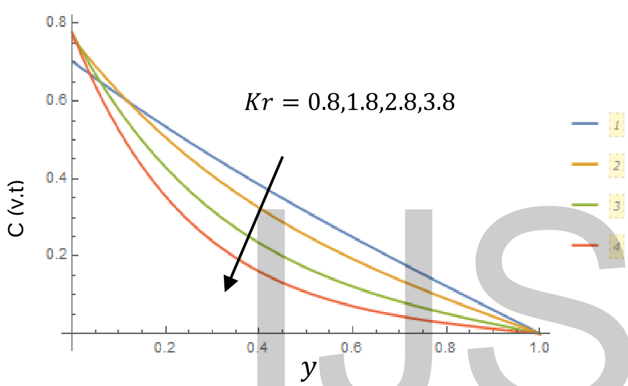

Figure 4.13 **Concentration Profile with Variation of Chemical Reaction Parameter Kr**
$L = 0.5, Sc = 1, \lambda = 0.5, t = 1$

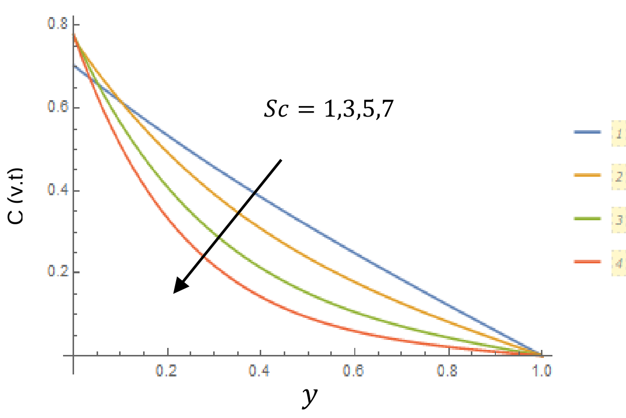

Figure 4.14 **Concentration Profile with Variation of Schmidt number Parameter Sc**
$Kr = 0.8, L = 0.5, \lambda = 0.5, t = 1$

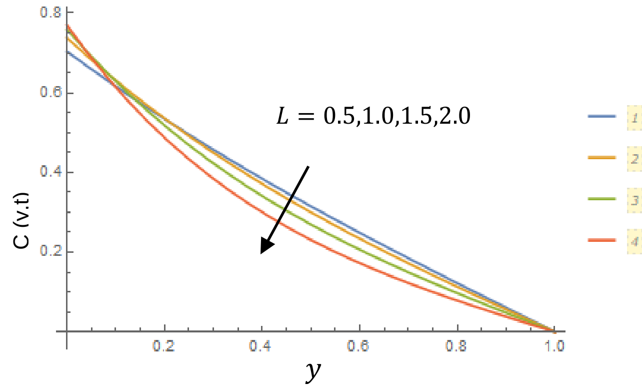

Figure 4.15 **Concentration Profile with Variation of Porous Medium diameter L**
$Kr = 0.8, Sc = 1, \lambda = 0.5, t = 1$

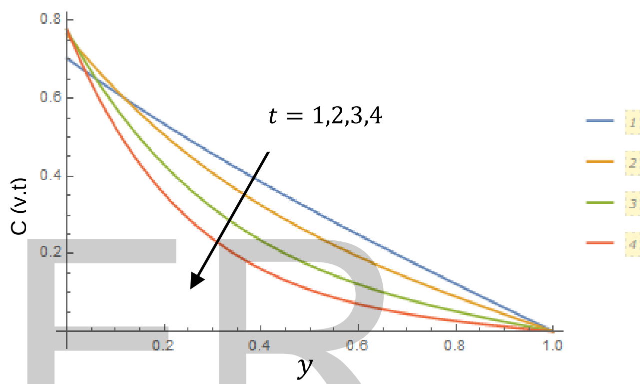

Figure 4.16 **Concentration Profile with Variation of time Parameter t**
$Kr = 0.8, L = 0.5, Sc = 1, \lambda = 0.5, t = 1$

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5. Conclusion

Chemical reaction, radiation and heat source effects on unsteady MHD blood flow over a horizontal porous surface in the presence of an inclined magnetic field and the effects of other parameters are summarized as follows,

- Increase in thermal radiation will increase the blood flow while the temperature of the blood reduces, which causes the cancerous cells to die in the region of exposure and will also shrink tumor.
- Increase in chemical reaction reduces the blood flow and increase the blood diffusion causing an increase in the concentration profile which kills the cancer cells.
- Increase in the Magnetic field introduces a Lorentz force which in turn increases viscosity and causes the blood flow to reduce. This process treats high blood pressure by reducing the blood pressure to a normal.
- Increase in heat source caused the blood flow to reduce and increases the temperature.

- Increase in porosity increases the blood flow and later has a mixed effect near the boundary layer.

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Appendix

$$b_1 = \frac{e^{m1}}{1-e^{3m1}};\; b_2 = \frac{e^{3m1}}{e^{3m1}-1};\, c_1 = \frac{e^{m2}}{1-e^{3m2}};\; c_2 = \frac{e^{3m2}}{e^{3m2}-1};$$

$$d_1 = \frac{-d_7+d_3e^{-m2}+d_4e^{m2}+d_5e^{-m1}+d_6e^{m1}-d_2e^{m3}}{e^{m3}};$$

$$d_1 = \frac{1-d_7\left[1-e^{-2m3}\right]-d_3[e^{-(m2+2m3)}-e^{m2}]-d_4[e^{(m2-2m3)}-e^{-m2}]-d_5[e^{-(m1+2m3)}-e^{m1}]+d_6[e^{(m1-2m3)}-e^{-m1}]}{e^{m3}-e^{-m3}}$$

$$d_3 = \frac{g\beta_T(e^{m2})}{(1-e^{3m2})(m_2^2+Lm_2+\sigma)};\; d_4 = \frac{g\beta_T(e^{3m2})}{(e^{3m2}-1)(m_2^2-Lm_2+\sigma)};$$

$$d_5 = \frac{g\beta_c(e^{m1})}{(1-e^{3m1})(m_1^2+Lm_1+\sigma)};$$

$$d_6 = \frac{g\beta_c(e^{3m1})}{(e^{3m1}-1)(m_1^2-Lm_1+\sigma)};\; \sigma^2 = \lambda^2 - M\cos^2\alpha + \frac{L^2}{K};$$

$$d_7 = \frac{h}{\sigma^2};$$

$$m_1 = \frac{1}{2}\left(ScL \pm \sqrt{Sc^2L^2 - 4Sc(\lambda^2 - Kr)}\right);$$

$$m_2 = \frac{1}{2(Pr+R)}\left(PrL \pm \sqrt{Pr^2L^2 - 4(Pr+R)(S+Pr\lambda^2)}\right);$$

$$m_3 = L \pm \sqrt{L^2 - 4\,\sigma^2};$$